‘Understanding’ cosmological bulk viscosity

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Abstract

A universe consisting of two interacting perfect fluids with the same 4-velocity is considered. A heuristic mean free time argument is used to show that the system as a whole cannot be perfect as well but neccessarily implies a nonvanishing bulk viscosity. A new formula for the latter is derived and compared with corresponding results of radiative hydrodynamics.

Key words: cosmology: theory - relativity - hydrodynamics - radiative transfer

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1 Introduction

In the realm of cosmology bulk viscosity is the most favorite dissipative phenomenon. Different from shear viscosity and heat conductivity, it is compatible with the symmetry requirements of the homogeneous and isotropic Friedmann Lemaître Robertson Walker (FLRW) universes. In the simplest cosmological models there is no way to study entropy producing processes except through bulk viscosity.

While this is obvious on formal grounds it is probably fair to say that the degree of 'understanding' bulk viscosity physically is by far less than that for shear viscosity or heat flux. Although the corresponding effect for a simple gas is known since the work of Israel (1963) and a radiative bulk viscosity coefficient was derived by Weinberg (1971) who also gave an analysis on its rôle in cosmology, followed by subsequent investigations of Straumann (1976) and Schweizer (1982), there remained the desire of some more intuitive insight beyond the involved calculations of radiative hydrodynamics. This problem was addressed in the last part of a paper by Udey & Israel (1982) who argued that for a two-fluid universe the mechanism responsible for bulk viscosity is a microscopic heat flux that compensates the temperature differences caused by different cooling rates of the two components. Their consideration of this point was based on the following semiquantitative argument. Let τ be the characteristic time for the interaction between both fluids such that during a time interval τ the perfect fluid components may be considered as effectively insulated from each other, resulting in different adiabatic cooling rates due to their different equations of state.

The present paper was inspired by this kind of arguing. While we shall confirm below that the difference in the cooling rates is indeed the essential point, we shall avoid in our investigation the introduction of microscopic heat fluxes. Although this concept may be helpful on small scales, it seems less convincing on large scales, e.g., of the Hubble scale. The microscopic gradients at different points had to conspire in order to produce a nonvanishing bulk viscosity in homogeneous and isotropic universes, which, however, is incompatible with the symmetry requirements of the latter. Moreover, the apparent reduction of a bulk viscous pressure to heat fluxes is not consistent with the fact that both phenomena are basically independent.

It is the aim of this paper is to show that *different cooling rates for two perfect fluids are sufficient for the existence of a nonvanishing bulk viscosity of the system as a whole.* No additional concept like that of a heat flux over intermolecular distances has to be used. The basic idea is to study a universe

of two different interacting perfect fluids and to ask for the conditions under which an effective one-fluid description is possible. It turns out that this one-fluid universe is neccessarily dissipative.

The present paper is semiquantitative throughout. We are not claiming to improve any of the technically rather complicated calculations in radiative hydrodynamics. Our objective is to achieve a kind of phenomenological 'understanding' of the bulk pressure phenomenon in the expanding Universe. On this level of description, we shall derive a new formula for the coefficient of bulk viscosity in a two-fluid system.

In section 2 the relevant relations for two noninteracting perfect fluids are presented and the corresponding cooling rates in an expanding universe are obtained. Section 3 is devoted to an effective one-fluid description for two fluids with mutual interaction. An explicit expression for the coefficient of bulk viscosity is derived with the help of a mean free time argument. The latter result is compared with work on radiative hydrodynamics in the context of relativistic kinetic theory in section 4. Section 5 gives a brief summary of the paper.

2 Two-fluid dynamics

The content of the Universe is assumed to be describable by an energy momentum tensor T^{ik} that is the sum of two different perfect fluid contributions which share the same 4-velocity:

$$T^{ik} = T_1^{ik} + T_2^{ik} \ , \tag{1}$$

with $(A = 1,\, 2)$

$$T_A^{ik} = \rho_A u^i u^k + p_A h^{ik} \ . \tag{2}$$

ρ_A is the energy density and p_A is the equilibrium pressure of species A. u^i is the common 4-velocity and h^{ik} is the projection tensor $h^{ik} = g^{ik} + u^i u^k$. Let us first deal with the case that the energy momentum conservation laws hold for each fluid separately:

$$T^{ik}_{A;k} = 0 \ , \tag{3}$$

implying the energy balances

$$\dot{\rho}_A = -\Theta \left(\rho_A + p_A\right) \ , \tag{4}$$

with the fluid expansion $\Theta \equiv u^i_{;i}$. $\dot{\rho}_A \equiv \rho_{A,i}u^i$ etc. Because of (3) both fluids evolve independently except for their mutal gravitational coupling. The particle flow vector N^i_A of species A is defined as

$$N^i_A = n_A u^i \ , \tag{5}$$

where n_A is the particle number density. Particle number conservation is expressed by $N^i_{A;i} = 0$, equivalent to

$$\dot{n}_A + \Theta n_A = 0 \ . \tag{6}$$

Let us further assume equations of state in the general form

$$p_A = p_A\left(n_A, T_A\right) \tag{7}$$

and

$$\rho_A = \rho_A\left(n_A, T_A\right) \ , \tag{8}$$

i.e., let particle number densities n_A and temperatures T_A be our basic thermodynamical variables. The temperatures of both components will be different in general.

Differentiating relation (8), using the balances (4) and (6) as well as the general relation

$$\frac{\partial \rho_A}{\partial n_A} = \frac{\rho_A + p_A}{n_A} - \frac{T_A}{n_A}\frac{\partial p_A}{\partial T_A} \ , \tag{9}$$

that follows from the requirement that the entropy is a state function, we find the following expression for the temperature behaviour:

$$\dot{T}_A = -T_A \Theta \frac{\partial p_A / \partial T_A}{\partial \rho_A / \partial T_A} \ . \tag{10}$$

It is obvious that the temperatures of both components behave differently for different equations of state. With $\Theta = 3\dot{R}/R$, where R is the scale factor of the Robertson-Walker metric, the equations of state $p_1 = n_1 k T_1$, $\rho_1 = 3 n_1 k T_1$ reproduce the well known $T_1 \sim R^{-1}$ behaviour for radiation. With $p_2 = n_2 k T_2$, $\rho_2 = n_2 m c^2 + \frac{3}{2} n_2 k T_2$ one obtains $T_2 \sim R^{-2}$ for matter.

3 Effective one-fluid dynamics

The hitherto independent fluids are now allowed to interact. We try to find an effective one fluid description for the Universe as a whole, characterized

by the particle number density $n = n_1 + n_2$ and an equilibrium temperature T. The overall equations of state are

$$p = p\left(n, T\right) \tag{11}$$

and

$$\rho = \rho\left(n, T\right) \ , \tag{12}$$

where p is the equilibrium pressure and ρ is the energy density of the system as a whole. The equilibrium temperature T is *defined* by (cf. Udey & Israel 1982)

$$\rho_1\left(n_1, T_1\right) + \rho_2\left(n_2, T_2\right) = \rho\left(n, T\right) \ . \tag{13}$$

As we shall show below this implies

$$p_1\left(n_1, T_1\right) + p_2\left(n_2, T_2\right) \neq p\left(n, T\right) \ . \tag{14}$$

For perfect fluids the difference between both sides of the latter inequality is the viscous pressure π [1]:

$$\pi = p_1\left(n_1, T_1\right) + p_2\left(n_2, T_2\right) - p\left(n, T\right) \ . \tag{15}$$

The existence of a nonvanishing viscous pressure is a consequence of the different temperature evolution laws of the subsystems. This is most easily understood by the following simple mean free time argument. Let τ be the characteristic mean free time for the interaction between both components. The time τ is assumed to be much larger than the characteristic interaction times within each of the components. Consequently, the latter may be regarded as perfect fluids on time scales of the order of τ. The interaction between the fluids is modelled by 'collisional' events, where τ plays the rôle of a mean free time between these 'collisions'. During the time interval τ, i.e., between subsequent interfluid interaction events, both components then evolve according their internal perfect fluid dynamics, given by (4), (6) and (10). Assume that through this interaction an element of the cosmic fluid is in equilibrium at a proper time η_0 at a temperature $T\left(\eta_0\right) = T_1\left(\eta_0\right) = T_2\left(\eta_0\right)$ with $p\left(\eta_0\right) = p_1\left(\eta_0\right) + p_2\left(\eta_0\right)$. Here, $p\left(\eta_0\right)$ and $p_A\left(\eta_0\right)$ are shorts for $p\left[n\left(\eta_0\right), T\left(\eta_0\right)\right]$ and $p_A\left[n_A\left(\eta_0\right), T_A\left(\eta_0\right)\right]$, respectively. Using the condition (13) at the proper time $\eta_0 + \tau$ up to first order in τ, i.e.,

$$\rho_A\left(\eta_0 + \tau\right) = \rho_A\left(\eta_0\right) + \tau\dot{\rho}_A\left(\eta_0\right) + ... \tag{16}$$

[1] For real fluids this is not neccessarily true (see, e.g. Kirkwood & Oppenheim 1961 chapter 7.7)

and

$$\rho(\eta_0 + \tau) = \rho(\eta_0) + \tau\dot{\rho}(\eta_0) + ... \tag{17}$$

where $\rho_A(\eta_0) \equiv \rho_A[n_A(\eta_0), T_A(\eta_0)]$ and $\rho(\eta_0) \equiv \rho[n(\eta_0), T(\eta_0)]$, applying (4) on the l.h.s. of (13) and the general relation

$$\frac{\partial\rho}{\partial n} = \frac{\rho + p}{n} - \frac{T}{n}\frac{\partial p}{\partial T} , \tag{18}$$

on its r.h.s., one finds

$$\dot{T}(\eta_0) = -T\frac{\partial p/\partial T}{\partial\rho/\partial T}\Theta \tag{19}$$

at the point $\eta = \eta_0$.

During the following time interval τ, i.e., until a subsequent 'collision', the subsystems move freely according to their proper dynamics given by (4), (6) and (10). At the time $\eta_0 + \tau$ we have $T(\eta_0 + \tau) \neq T_1(\eta_0 + \tau) \neq T_2(\eta_0 + \tau) \neq T(\eta_0 + \tau)$ in general. With

$$T_A(\eta_0 + \tau) = T_A(\eta_0) + \tau\dot{T}_A(\eta_0) + ... \tag{20}$$

and

$$T(\eta_0 + \tau) = T(\eta_0) + \tau\dot{T}(\eta_0) + ... \tag{21}$$

we find from (10) and (19)

$$T_1 - T_2 = -\tau\Theta T\left(\frac{\partial p_1/\partial T}{\partial\rho_1/\partial T} - \frac{\partial p_2/\partial T}{\partial\rho_2/\partial T}\right) , \tag{22}$$

$$T_1 - T = -\tau\Theta T\left(\frac{\partial p_1/\partial T}{\partial\rho_1/\partial T} - \frac{\partial p/\partial T}{\partial\rho/\partial T}\right) , \tag{23}$$

$$T_2 - T = -\tau\Theta T\left(\frac{\partial p_2/\partial T}{\partial\rho_2/\partial T} - \frac{\partial p/\partial T}{\partial\rho/\partial T}\right) , \tag{24}$$

up to first order in τ. Due to the different cooling rates (10) and (19) there occur temperature differences at any point of the expanding fluid: Both differences in the temperatures of the components, i.e., between T_1 and T_2, and differences between the temperature of each of the components and the temperature T of the system as a whole.

In order to arrive at our conclusion $\pi \neq 0$ one has simply to consider the sum of the partial pressures p_1 and p_2 at $\eta = \eta_0 + \tau$ up to first order in τ. The latter may be written as

$$\begin{aligned} p_1(n_1, T_1) + p_2(n_2, T_2) &= p_1(n_1, T) + p_2(n_2, T) \\ &\quad + (T_1 - T)\frac{\partial p_1}{\partial T} + (T_2 - T)\frac{\partial p_2}{\partial T} . \end{aligned} \tag{25}$$

Inserting the temperature differences $T_1 - T$ and $T_2 - T$ at $\eta_0 + \tau$ from (23) and (24), we find

$$\begin{aligned} p_1(n_1, T_1) + p_2(n_2, T_2) &= p(n, T) - \tau T \Theta \left[\frac{\partial p_1}{\partial T} \left(\frac{\partial p_1 / \partial T}{\partial \rho_1 / \partial T} - \frac{\partial p / \partial T}{\partial \rho / \partial T} \right) \right. \\ &\quad \left. + \frac{\partial p_2}{\partial T} \left(\frac{\partial p_2 / \partial T}{\partial \rho_2 / \partial T} - \frac{\partial p / \partial T}{\partial \rho / \partial T} \right) \right] , \end{aligned} \tag{26}$$

where $p(n, T) \equiv p_1(n_1, T) + p_2(n_2, T)$ was used. Applying the zeroth-order relations $\partial p / \partial T = \partial p_1 / \partial T + \partial p_2 / \partial T$ and $\partial \rho / \partial T = \partial \rho_1 / \partial T + \partial \rho_2 / \partial T$ in the bracket on the r.h.s. of (26) one gets, after a simple rearrangement, the first-order result

$$p_1(n_1, T_1) + p_2(n_2, T_2) = p(n, T) + \pi , \tag{27}$$

where

$$\pi \equiv \tau T \Theta \frac{\partial \rho}{\partial T} \left(\frac{\partial p_1 / \partial T}{\partial \rho_1 / \partial T} - \frac{\partial p / \partial T}{\partial \rho / \partial T} \right) \left(\frac{\partial p_2 / \partial T}{\partial \rho_2 / \partial T} - \frac{\partial p / \partial T}{\partial \rho / \partial T} \right) \tag{28}$$

is generally different from zero. This proves our initial statement that a system of two interacting perfect fluids is not perfect as well. While the energy momentum tensors of the subsystems are given by (2), the system as a whole is characterized by

$$T^{ik} = \rho u^i u^k + (p + \pi) h^{ik} . \tag{29}$$

To separate bulk viscosity from any other dissipative phenomenon we have ignored here the possibility of nonvanishing heat fluxes and shear stresses in inhomogeneous and anisotropic cosmological models.

From the definition

$$\pi = -\zeta \Theta , \tag{30}$$

of the bulk viscosity ζ, the latter is found to be given by

$$\zeta = -\tau T \frac{\partial \rho}{\partial T} \left(\frac{\partial p_1}{\partial \rho_1} - \frac{\partial p}{\partial \rho} \right) \left(\frac{\partial p_2}{\partial \rho_2} - \frac{\partial p}{\partial \rho} \right) , \tag{31}$$

where

$$\frac{\partial p_A}{\partial \rho_A} \equiv \left(\frac{\partial p_A}{\partial \rho_A} \right)_{n_A} \equiv \frac{\partial p_A / \partial T}{\partial \rho_A / \partial T} , \; \frac{\partial p}{\partial \rho} \equiv \left(\frac{\partial p}{\partial \rho} \right)_n \equiv \frac{\partial p / \partial T}{\partial \rho / \partial T} . \tag{32}$$

For 'ordinary' matter $\partial p_A/\partial \rho_A$ lies in the range $1/3 \leq \partial p_A/\partial \rho_A \leq 2/3$ (see the equations of state below (10)). The lower limit corresponds to radiation, the upper one to matter. $\partial p/\partial \rho$ will take a value intermediate between $\partial p_1/\partial \rho_1$ and $\partial p_2/\partial \rho_2$. Assuming without loss of generality $\partial p/\partial \rho > \partial p_1/\partial \rho_1$, we shall have $\partial p/\partial \rho < \partial p_2/\partial \rho_2$. Consequently, $\zeta \geq 0$, i.e., the entropy production is positive which agrees with the second law of thermodynamics (see, e.g., de Groot, van Leeuwen & van Weert 1980).

To the best of our knowledge the expression (31) for the coefficient of bulk viscosity is a new result . Although based on heuristic arguments its structure is rather general. Formula (31) is valid for general equations of state (7), (8) and (11), (12). Until now we did not specify to the case that one of the components obeys the equations of state for radiation.

4 Relation to radiative hydrodynamics

The expression (31) for the coefficient of bulk viscosity is similar but not identical to the corresponding expressions of radiative hydrodynamics found by Weinberg (1971), Straumann (1976), Schweizer (1982), Udey & Israel (1982) and Pavón, Jou & Casas-Vázquez (1983). The reason for this is a physical one and not due to the semiquantitative nature of our considerations. Both components of the system are treated as fluids with different equations of state in our setting. The final result (31) is therefore symmetric under a change of the labels 1 and 2 that identify both components. In the work of Weinberg (1971), Straumann (1976), Schweizer (1982), Udey & Israel (1982) and Pavón, Jou & Casas-Vázquez (1983) on the other hand, both components are treated asymmetrically. While one of the components is a fluid as well, the second one, a radiation component, is described with the help of kinetic theory. The main asymmetry lies in the assumption of the mentioned authors that the radiation component is allowed to deviate from local equilibrium while the fluid component is not. Of course, the result for the coefficient of bulk viscosity is not symmetric in the components either and a coincidence with (31) cannot be expected.

It might be useful to compare some of the basic relations of radiative hydrodynamics with our framework. For definitenes, let component 1 of our analysis be the radiation and component 2 the material component. Since there does not appear a separate radiation temperature in the papers by Weinberg (1971), Straumann (1976), Schweizer (1982), Udey & Israel (1982) and Pavón, Jou & Casas-Vázquez (1983), let us eliminate the latter

in (13). With $\rho_1(n_1, T_1) = \rho_1(n_1, T_2) + (T_1 - T_2)\,\partial\rho_1/\partial T + ...$ we find, up to first order in the temperature difference,

$$\rho(n, T) = \rho_1(n_1, T_2) + \rho_2(n_2, T_2) + \hat{\rho}\ , \tag{33}$$

where

$$\hat{\rho} \equiv (T_1 - T_2)\frac{\partial\rho_1}{\partial T}\ . \tag{34}$$

Using (22) with (32) we have

$$\hat{\rho} = -\tau T\Theta\frac{\partial\rho_1}{\partial T}\left(\frac{\partial p_1}{\partial\rho_1} - \frac{\partial p_2}{\partial\rho_2}\right)\ . \tag{35}$$

For comparison with the results of Udey & Israel (1982) for radiative hydrodynamics it is helpful to rewrite the latter expression as

$$\hat{\rho} = \hat{\rho}_{UI} + \tau T\Theta\frac{\partial\rho_1}{\partial T}\left(\frac{\partial p_2}{\partial\rho_2} - \frac{\partial p}{\partial\rho}\right)\ , \tag{36}$$

where

$$\hat{\rho}_{UI} = -\tau T\Theta\frac{\partial\rho_1}{\partial T}\left(\frac{\partial p_1}{\partial\rho_1} - \frac{\partial p}{\partial\rho}\right)\ . \tag{37}$$

With (36) and (37) relation (33) may be compared with formula (31) of Udey & Israel (1982), which, in our notation, reads

$$\rho(n, T) = \rho(n, T_2) + aT^4B\ . \tag{38}$$

The quantity $\hat{\rho}$ in our equation (33) is the counterpart of the term aT^4B in Udey & Israel (1982), which describes the deviation of the radiation component from equilibrium. With $T\partial\rho_1/\partial T = 4aT^4$ and $\partial p_1/\partial\rho_1 = 1/3$ for radiation and specification of (35b), (45) and (47) in Udey & Israel (1982) to the Eckart case (i.e., neglecting relaxation and cross effects), we find that aT^4B in (38) (equation (31) in Udey & Israel 1982) coincides with $\hat{\rho}_{UI}$ of our equation (37). The circumstance that there exists a difference between $\hat{\rho}$ and $\hat{\rho}_{UI}$ reflects the above mentioned fact that, different from radiative hydrodynamics, in our setting both components are allowed to deviate from equilibrium. For $\partial p_2/\partial\rho_2 \approx \partial p/\partial\rho$ this difference becomes negligible and we have $\hat{\rho} \approx \hat{\rho}_{UI}$.

A similar statement holds for the difference of the temperatures $T_2 - T$. By virtue of the zeroth-order identity

$$\frac{\partial\rho}{\partial T}\left(\frac{\partial p}{\partial\rho} - \frac{\partial p_2}{\partial\rho_2}\right) = \frac{\partial\rho_1}{\partial T}\left(\frac{\partial p_1}{\partial\rho_1} - \frac{\partial p_2}{\partial\rho_2}\right) \tag{39}$$

the temperature difference (24) may be written as

$$\begin{aligned} T - T_2 &= -\tau T \Theta \frac{\partial \rho_1/\partial T}{\partial \rho/\partial T} \left(\frac{\partial p_1}{\partial \rho_1} - \frac{\partial p_2}{\partial \rho_2} \right) \\ &= (T - T_2)_W + \tau T \Theta \frac{\partial \rho_1/\partial T}{\partial \rho/\partial T} \left(\frac{\partial p_2}{\partial \rho_2} - \frac{\partial p}{\partial \rho} \right) \ , \end{aligned} \tag{40}$$

where

$$(T - T_2)_W = -\tau T \Theta \frac{\partial \rho_1/\partial T}{\partial \rho/\partial T} \left(\frac{\partial p_1}{\partial \rho_1} - \frac{\partial p}{\partial \rho} \right) \ , \tag{41}$$

if specified to radiation, coincides with Weinberg's relation (2.38) (Weinberg 1971) for radiative hydrodynamics. The condition for $T - T_2 \approx (T - T_2)_W$ is again $\partial p_2/\partial \rho_2 \approx \partial p/\partial \rho$.

Using the corresponding intermediate steps that led to (33) now for the sum of the pressures, we get the first-order relation

$$p_1 (n_1, T_1) + p_2 (n_2, T_2) = p_1 (n_1, T_2) + p_2 (n_2, T_2) + \hat{p} \ , \tag{42}$$

with

$$\hat{p} \equiv (T_1 - T_2) \frac{\partial p_1}{\partial T} = \frac{\partial p_1}{\partial \rho_1} \hat{\rho} \ . \tag{43}$$

For the first two terms on the r.h.s. of (42) we may write, again up to first order,

$$p_1 (n_1, T_2) + p_2 (n_2, T_2) \equiv p (n, T_2) = p (n, T) + (T_2 - T) \frac{\partial p}{\partial T} \ . \tag{44}$$

From (40) and (35) one has

$$T_2 - T = - \left(\frac{\partial \rho}{\partial T} \right)^{-1} \hat{\rho} \ , \tag{45}$$

and, consequently,

$$p_1 (n_1, T_1) + p_2 (n_2, T_2) = p (n, T) + \pi \ , \tag{46}$$

with

$$\pi = \left(\frac{\partial p_1}{\partial \rho_1} - \frac{\partial p}{\partial \rho} \right) \hat{\rho} \ . \tag{47}$$

This is relation (33) of Udey & Israel (1982).

In order to avoid misunderstandings we point out that it was not necessary for our derivation of formula (31) to introduce the quantities $\hat{\rho}$ and

$\hat{p}$. These quantities are useful, however, for the comparison with work done in radiative hydrodynamics.

By virtue of the identity (39), ζ from (31) may be written as

$$\zeta = \tau T \frac{\partial \rho_1}{\partial T} \left(\frac{\partial p_1}{\partial \rho_1} - \frac{\partial p_2}{\partial \rho_2} \right) \left(\frac{\partial p_1}{\partial \rho_1} - \frac{\partial p}{\partial \rho} \right) \ . \tag{48}$$

While it is generally not to be expected that this expression coincides with Weinberg's coefficient (Weinberg 1971)

$$\zeta_W = \tau T \frac{\partial \rho_1}{\partial T} \left(\frac{\partial p_1}{\partial \rho_1} - \frac{\partial p}{\partial \rho} \right)^2 \ , \tag{49}$$

it is obvious that both results are the closer the better the approximation of $\partial p/\partial \rho$ by $\partial p_2/\partial \rho_2$ in (31) will be. In other words, the results (48) and (49) are similar if the matter component dominates the behaviour of the system as a whole.

The bulk viscosity coefficients ζ and ζ_W are related by

$$\zeta = \zeta_W + \tau T \frac{\partial \rho_1}{\partial T} \left(\frac{\partial p}{\partial \rho} - \frac{\partial p_2}{\partial \rho_2} \right) \left(\frac{\partial p_1}{\partial \rho_1} - \frac{\partial p}{\partial \rho} \right) \ . \tag{50}$$

Using (31) on the r.h.s of the latter equation we obtain the following relation between ζ and ζ_W:

$$\frac{\partial \rho_2}{\partial \rho} \zeta = \zeta_W \ , \tag{51}$$

where

$$\frac{\partial \rho_2}{\partial \rho} \equiv \frac{\partial \rho_2 / \partial T}{\partial \rho / \partial T} \ . \tag{52}$$

Relation (51) shows again that the generally different expressions for ζ and ζ_W will become similar if the overall behaviour of the system is determined by the fluid component 2.

For the specific case of a mixture of radiation and matter with equations of state $p_1 = n_1 k T_1$, $\rho_1 = 3 n_1 k T_1$, $p_2 = n_2 k T_2$, $\rho_2 = n_2 m c^2 + \frac{3}{2} n_2 k T_2$ the bulk viscosity coefficient (31) reduces to

$$\zeta = \frac{\tau}{3} n_1 k T \frac{n_2}{2 n_1 + n_2} \ . \tag{53}$$

Since $\partial \rho_2 / \partial \rho = n_2 / \left(2 n_1 + n_2\right)$ one has $\zeta \approx \zeta_W$ for $n_1 \ll n_2$. The lower the ratio n_1/n_2 of the photon number density to the number density of the matter particles the closer to unity is the ratio ζ_W / ζ.

A fluid description of the Universe makes sense as long as $\tau \ll H^{-1}$, where $H \equiv \Theta/3$ is the Hubble parameter. Combining (53) with (30) we find that this condition is consistent with $\mid \pi \mid \ll p_A$, i.e., the magnitude of the nonequilibrium part π of the pressure is much smaller than the equilibrium pressures as it is neccessary for a first-order approach like that of the present paper to be valid.

5 Summary

This paper is a heuristic attempt to clarify the origin of bulk viscosity in the expanding universe. Characterizing the interaction between two different fluids, each of them perfect on its own, by an effective mean free time parameter τ and assuming a free evolution of both components according to their internal perfect fluid dynamics during the time interval τ, i.e., between subsequent interfluid interaction events, the different cooling rates (due to different equations of state) of the components lead to a nonvanishing bulk pressure of the system as a whole. A new formula for the coefficient of bulk viscosity of a two-fluid mixture was obtained and the relation of this expression to the results of radiative hydrodynamics was clarified.

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